

A NOTE ON THE EFFECT OF AFFINITY HAEMODIALYSIS ON T-CELL DEPLETION

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ABSTRACT

We modify existing mathematical models for HIV that account for observation from haemodialysis. Of particular interest are the criteria under which the disease infected equilibrium could be stable we indicate treatment that is adequate to significantly lower gp 120 levels and help T cells to recover to normal level.

KEYWORDS: Affinity Dialysis, HIV/AIDS, Enveloped Protein gp 120, Criteria for Stability

1. INTRODUCTION

1.1 What is Known

The hallmark of HIV disease is the gradual loss of CD4⁺T cells, which ultimately leaves the immune system unable to defend against opportunistic infections. While the mechanism through which HIV causes AIDS is imperfectly understood, the clinical data suggest that HIV derived envelope proteins appear to be intimately involved. The major HIV envelope glycoprotein gp 120 has been shown to have profound biological effects in vitro. gp 120 causes CD4+ T cells to undergo apoptosis. Binding o gp 120 to CD4+T cells in the presence of anti-envelope antibodies and compliment opsonizes the cells, targeting them for clearance. The combine effect is the destruction of uninfected immune cells [1,2,3,4,5,6,7 and 8].

1.2 Model for AIDS Incorporating Stem Cell Depletion and gp 120 Biological Effects

Table 1.1

Production	
$S \xrightarrow{k_1} T :$	T- cell production from stem cells
$T + V \xrightarrow{k_2} T_i :$	Infection of T- cell
$T_i \xrightarrow{k_3} V + P :$	Production of virus and gp 120
$P + T \xrightleftharpoons{k_4} PT :$	Reversible gp 120 binding to normal T-cells
Clearance	
$T \xrightarrow{d_1} :$	Death rate of normal T cell
$T_i \xrightarrow{d_2} :$	Death clearance of infected T cells
$V \xrightarrow{d_3} :$	Clearance rate of virus
$P \xrightarrow{d_4} :$	Clearance gp 24
$PT \xrightarrow{d_5} :$	Clearance gp 120 induced apoptosis and clearance

Where T denotes healthy cell, T_i is the infected T cells, V is the virus, p is the concentration of gp 24.

We further assume that λ is the rate of recovery of T_i .

2. MATHEMATICAL MODEL

Arising from table 1.1, the approximate mathematical equations are

$$\frac{dS}{dt} = -k_1 S \left(1 - \frac{T}{T_0} \right) \quad (2.1)$$

$$\frac{dT}{dt} = k_1 S \left(1 - \frac{T}{T_0} \right) - k_2 TV - d_1 T - d_5 \frac{10PT}{k_4} + \lambda T_i \quad (2.2)$$

$$\frac{dT_i}{dt} = k_2 TV - d_2 T_i \quad (2.3)$$

$$\frac{dV}{dt} = k_3 T_i - d_3 V \quad (2.4)$$

$$\frac{dP}{dt} = 0.777V - d_4 P \quad (2.5)$$

The present model essentially modifies [7] model assuming that some T_i essentially recover and become healthy even weakly treatment of dialysis would be adequate to significantly lower gp 120 levels and help T cells recover to normal levels as long as stem cells remains available.

3. STABILITY ANALYSIS

The critical points are $(0, 0, 0, 0, 0)$ and $\left(S_0, T_0, \frac{d_1 d_4 k_3 T_o^2}{0.777\alpha}, \frac{d_1 d_4 T_o}{0.777\alpha}, \frac{d_1 T_o}{\alpha} \right)$,

where $\alpha = (\lambda - d_2) \frac{d_3 d_4}{k_3 (0.777)} - \frac{d_3 10 T_0}{k_4}$. The point $(0, 0, 0, 0, 0)$ is the disease free equilibrium.

The point $\left(S_0, T_0, \frac{d_1 d_4 k_3 T_o^2}{0.777\alpha}, \frac{d_1 d_4 T_o}{0.777\alpha}, \frac{d_1 T_o}{\alpha} \right) = (S_0, T_0, a, b, c)$ is the infected equilibrium.

Let $\phi = S - S_0, \psi = T - T_0, x = T_i - a, y = v - b, z = p - c$. Then

$$\frac{d\phi}{dt} = -\frac{S_0 \psi}{T_0} + \text{non linear terms} \quad (3.1)$$

$$\frac{d\psi}{dt} = \frac{S_0 \psi}{T_0} - k_2 T_0 y - k_2 b \psi - d_1 \psi - \frac{10 d_5 c \psi}{k_4} - \frac{10 d_5 c T_0}{k_4} + \lambda x + \text{non linear terms} \quad (3.2)$$

$$\frac{dx}{dt} = k_2 T_o y + k_2 b \psi - d_2 x \quad (3.3)$$

$$\frac{dy}{dt} = k_3 x - d_3 y \quad (3.4)$$

$$\frac{dz}{dt} = 0.777 y - d_4 z \quad (3.5)$$

So the relevant matrix is

$$\begin{pmatrix} 0 & -\frac{S_o}{T_o} & 0 & 0 & 0 \\ 0 & 0 & \lambda & -k_2 T_o & -\frac{10d_5 T_o}{k_4} \\ 0 & 0 & -d_2 & k_2 T_o & 0 \\ 0 & 0 & k_3 & -d_3 & 0 \\ 0 & 0 & 0 & 0.777 & -d_4 \end{pmatrix}$$

The eigenvalue μ satisfies $\mu^2 (d_4 + \mu) ((d_2 + \mu)(d_3 + \mu) - k_3 k_2 T_o) = 0$

Hence $\mu = 0$ twice or $\mu_1 = \mu_2 = 0$ $\mu_3 = -d_4$

$$\mu_4 = \left\{ \frac{-(d_2 + d_3) - \sqrt{(d_2 + d_3)^2 + 4k_2 k_3 T_o}}{2} \right\}$$

$$\mu_5 = \left\{ \frac{-(d_2 + d_3) + \sqrt{(d_2 + d_3)^2 + 4k_2 k_3 T_o}}{2} \right\}$$

4. DISCUSSIONS OF RESULTS

The infected equilibrium is unstable since some eigenvalues non- negative real values and may lead to full blow AIDS. To ensure stability of the infected point, it is necessary to ensure that the stem cell is not depleted. This is the subject of another paper.

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